ELECTROMAGNETISM

Faraday's law of electromagnetic induction:

Faraday stated two laws from the observations of Oersted study is called Faraday's law of electromagnetic induction

> **1) Whenever the magnetic flux linked with an electric circuit (coil) changes, an e.m.f is induced in the circuit (coil). The induced e.m.f exists as long as the change in magnetic flux continues.**

> 2) **The magnitude of induced e.m.f is directly proportional to the negative rate of variation of the magnetic flux linked with the circuit.**

If Φ_B be the magnetic flux linked with circuit at any instant and e be the induced e.m.f then

$$e = -\frac{d\Phi_B}{dt} \quad \rightarrow (1)$$

If there are N turns in the coil, then

$$e = -N\frac{d\Phi_B}{dt} \quad \rightarrow \quad (2)$$

The negative sign is in accordance with Lenz's law, this is also called **Neumann's Law**

Lenz's Law:

The Lenz's law is based on the principle of conservation of energy. Thus it helps to explain the direction (polarity) of induced e.m.f or induced current in a coil.

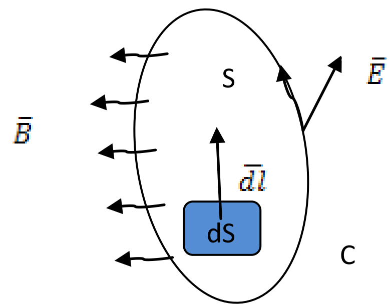
> **The polarity of the induced e.m.f is always such that it tends to produce a current which opposes the change in magnetic flux that produced it.**

The changing magnetic field and magnetic flux induces an electric current in a coil. This induced current itself creates magnetic field and hence magnetic flux is induced around the coil. Therefore the change in the external magnetic field and flux is always opposed.

Vector form of Faraday's law(Integral and Differential forms):

Integral form:

Consider that magnetic field is produced by a stationary magnet or current carrying coil. Suppose there is a closed circuit C of any shape which encloses a surface S in the field as shown in fig., Let $\bar{B}$ be the magnetic flux density in the neighborhood of the circuit.



The magnetic flux through a small area "dS" will be $\bar{B}$. Now the flux through the entire circuit is

$$\Phi_B = \int_S \bar{B}\,.dS \quad \rightarrow (1)$$

When the magnetic flux is changed, an electric field $\bar{E}$ induced around the circuit. The line integral of the electric field gives the induced e.m.f in the closed circuit. Thus

$$e = \oint E.dl \quad \rightarrow (2)$$

Where E is the electric field at an element of dl of the circuit. Substituting the values of e and Φ_B from equations (2) and (1) we have

$$\oint E.dl = -\frac{d\Phi_B}{dt} = = -\frac{d}{dt}\int_S B.ds \quad \rightarrow (3)$$

This is the integral form of Faraday's law.

Differential form:

According to equation (3) the line integral of the electric field around any closed circuit is equal to the negative rate of change of magnetic flux through the circuit.

Further by stokes theorem we have

$$\oint E.dl = \int_S (\nabla\times E).dS \quad \rightarrow (4)$$

From equation (3) and (4) we get

$$\int_S (\nabla\times E).dS \quad = -\frac{d}{dt}\int_S B.ds$$

Hence $\nabla\times E = -\frac{dB}{dt}$

This is the differential form of Faraday's law.

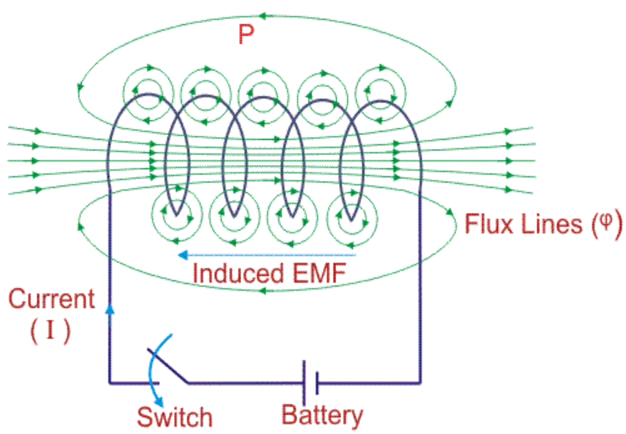
Self Induction:

The phenomenon of self induction was discovered by J. Henry in 1932. When a transient current passes through a coil it produces magnetic field around it, flux due to this field is linked with the coil itself. Due to its own flux change, an e.m.f is induced in the coil and it is called induced e.m.f or back e.m.f.

Definition: Self induction is the property of a coil by virtue of which opposes the growth or decay of the current flowing through it.

(OR)

Self inductance is the phenomenon of inducing e.m.f in a coil due to flow of current which changes with time in the same coil



Consider a coil connected to a battery through key (k). When the key is closed, due to increasing current in the coil, the magnetic field and hence flux linkage with the coil also increases. As a result of this, induced emf is set up in the coil. According to Lenz's law, the direction of induced emf is such that it opposes the growth of current in the coil. This delays the current to acquire the maximum value.

When the key is released, the current in the coil starts decreasing, so the magnetic flux linked with the coil decreases. As a result of this change in the magnetic flux, induced emf is set up in the coil itself. According to Lenz's law, the direction of induced emf is such that it opposes the decay of the current in the coil. This delays the current to acquire minimum (or) zero value.

NOTE: Self induction is also known as inertia of electricity as it opposes the growth or decay of the current in the circuit

Coefficient of self induction (or) self inductance:

The total magnetic flux Φ_B linked with the coil is proportional to the current I flowing in it i.e.,

$$\Phi_B \propto i$$

Or

$$\boxed{\Phi_B = Li} \quad \rightarrow (1)$$

Where L is a constant called the coefficient of self induction or self inductance of the coil. When i =1 , Φ_B =L. Hence the coefficient of self induction is numerically equal to the magnetic flux linked with the coil when unit current flows through it.

The e.m.f. induced in the coil is given by

$$e = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d(Li)}{dt}$$

$$= -L\frac{di}{dt} \quad \rightarrow (2)$$

The negative sign indicates that the induced e.m.f is in such a direction as to oppose the change

When $\frac{di}{dt}$ =1 ; e= -L

Therefore the coefficient of self inductance is numerically equal to the induced e.m.f in the coil, when the rate of change of current is unity.

Unit: The unit of self inductance is henry which is inductance of a coil in which an e.m.f of 1 volt is set up by the change of current at 1 ampere per second

$$1 \text{ henry} = \frac{1\ volt}{1\ amp/sec}$$

Energy stored in electric fields:

Consider a capacitor of capacitance C and carrying a charge q at any instant. Let the potential difference between the plates V, then

$$V = q/c \quad \rightarrow (1)$$

If an additional charge dq is to be given to this capacitor then some work must be done against the potential difference.

So the work done increasing charge by dq is given by

$$dW = V\,dq = (q/C)\,dq \quad \rightarrow (2)$$

∴ Total work to charge a capacitor to a charge q_0

$$W = \int dW = \int_0^{q_0} \frac{q}{C}\,dq = \frac{{q_0}^2}{2C} \quad \rightarrow (3)$$

Now the energy stored by a charged capacitor

$$U = W = \frac{1}{2}\frac{q^2}{C} = ½(CV^2) \quad (\because q = CV) \quad \rightarrow (4)$$

For a parallel plate capacitor of area A and plate separation d, the capacitance C is given by

$$C = \frac{\varepsilon_0 A}{d} \quad \text{and } V = Ed$$

$$\therefore \text{ Energy stored } U = \frac{1}{2} \times \frac{\varepsilon_0 A}{d} \times E^2 d^2$$

$$U = ½\,(\varepsilon_0 E^2 Ad) \text{ joules}$$

Energy stored in magnetic fields:

When the current in a coil is switched on, self induction opposes the growth of current i.e. the current flows against back e.m.f and does work against it.

$$dW = -\,e\,i\,dt$$

$$dW = +L\frac{di}{dt}\,i\,dt \qquad (\because e = -L\frac{di}{dt})$$

Hence, total work done in bringing the current from zero to a steady maximum value i_0 is

$$W = L\int_0^{i_0} i\, di$$

$$W = ½ (Li_0^2)$$

Consider a very long solenoid of length 'l' and cross-sectional area 'A'. When current flows in it, a magnetic field is established and the work done is stored as energy in the magnetic field given by

$$U = ½ (Li_0^2)$$

But inductance in coil is given by $L = \mu_0 n^2 Al$

Where n is number of turns in solenoid per meter.

$$\therefore U = ½ (\mu_0 n^2 Al\, i_0^2) = \frac{1}{2} \times \frac{(\mu_0 n i_0)^2}{\mu_0} \times Al$$

But the magnetic field inside a coil $B = \mu_0 n i_0$

$$U = \frac{B^2}{2\mu_0} \times Al$$

OR

$$U = \frac{\mu_0 H^2}{2} \times Al$$

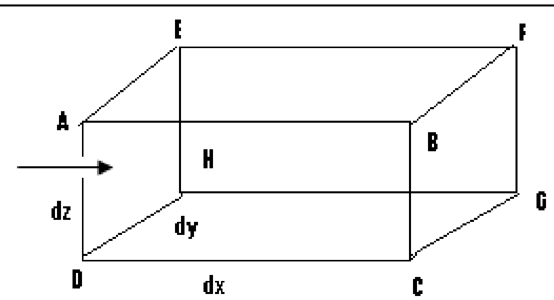
Poynting Vector:

One important characteristic of electromagnetic waves is that they transport energy from one point to another point.

The amount of field energy passing through unit area of the surface perpendicular to the direction of propagation of energy is called as Poynting vector.

The Poynting vector is denoted by $\bar{P}$ given by

$$\bar{P} = \frac{1}{\mu_0}(\bar{E} \times \bar{B}) \quad \text{or} \quad (\bar{E} \times \bar{H})$$



Proof: Consider an elementary volume in the form of a rectangular parallopiped of sides dx, dy and dz as shown in fig. The volume of parallopiped is "dxdydz". Suppose the energy is propagated along X-axis , now the area perpendicular to the direction of propagation of energy is dydz. Let the electromagnetic energy in this volume is U. Then the rate of change of energy is $\frac{\partial U}{\partial t}$.

$$\frac{\partial U}{\partial t} = -\oint_S P.ds \quad \rightarrow (1)$$

Negative sign is used to show that energy is entering in the volume.

Since the energy per unit volume in electric magnetic fields

$$U = (\tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2}\mu_0 H^2) \quad \rightarrow (2)$$

The rate of decrease of energy in volume dV is given b

$$-\frac{\partial U}{\partial t} = -\frac{\partial}{\partial t}\int_V (\tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2}\mu_0 H^2)\, dV \quad \rightarrow (3)$$

$$= \int_V -\left[\varepsilon_0 E(\frac{\partial E}{\partial t}) + \mu_0 H(\frac{\partial H}{\partial t})\right] dV \quad \rightarrow (4)$$

From Maxwell's equation

$$\frac{\partial E}{\partial t} = \frac{\nabla \times H}{\epsilon_0} \quad \text{and} \quad \frac{\partial H}{\partial t} = -\frac{\nabla \times E}{\mu_0} \quad \rightarrow (5)$$

From equation (5) and (4)

$$-\frac{\partial U}{\partial t} = \int_V [\, H.(\nabla \times E) - E.(\nabla \times H)]\, dV \quad \rightarrow (6)$$

$$= \int_V \nabla.(E \times H) dV \quad \rightarrow (7)$$

From Gauss theorem of divergence (7) can be written as

$$= \oint_S (E \times H) \bullet \hat{n} ds \quad \rightarrow (8)$$

Comparing (8) and (1)

$$\bar{P} = \bar{E} \times \bar{H}$$

The vector shows that energy flow takes place in a direction perpendicular to the plane containing E and H

Characteristics of Poynting Vector

(1) The poynting vector is perpendicular to both electric field vector E and magnetic field vector H
(2) The quantity∇. P represents the net energy flow in electromagnetic field
(3) In alternating fields

$$P_{avg} = E_{rms} \times H_{rms}$$

Fundamental laws of electromagnetism:

(1) Gauss law in electrostatics:

The electric flux Φ_E through a closed surface is equal to (1/ε) times the net charge q enclosed by the surface. i.e. The surface integral of the normal component of the electric field $\bar{E}$ over any closed surface equals (1/ε times) the net charge with that volume

$$\Phi_E = \oint \bar{E}\ ds = q/\varepsilon$$

(2) Gauss law in magnetostatics:

The magnetic lines of force leaving from the closed surface will be equal to the number of lines of force entering the surface. i.e. The net outward magnetic flux from any closed surface is zero.

$$\Phi_B = \oint \bar{B}\ ds = 0$$

(3) Faraday's law of electromagnetism

The magnitude of induced e.m.f is directly proportional to the negative rate of variation of the magnetic flux linked with the circuit.

$$\text{e.m.f. (e)} = -\frac{d\phi_B}{dt} \quad \text{(OR)} \quad \oint \bar{E}\cdot dl = -\int_s \frac{\partial B}{\partial t}\bullet ds$$

(4) Ampere's circuit law(or) Biot-Savart's law:

The steady current carrying conductor generates magnetic field around it

$$\oint \bar{B}\cdot dl = \mu_0 i$$

Basic equations of electromagnetism- Maxwell's equation

Maxwell's Law	Integral form	Differential form
First law (based on Gauss law of electrostatics)	$\oint \nabla \cdot \bar{D}\ ds = \int_V \rho \bullet dV$	$\nabla \cdot \bar{D} = \rho$
Second law (based on Gauss law of magnetostatics)	$\oint \bar{B}\ ds = 0$	$\nabla \cdot \bar{B} = 0$
Third law (based on Faraday's law of electromagnetic induction)	$\oint \bar{E} \cdot dl = -\int_S \frac{\partial B}{\partial t} \bullet ds$	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$
Fourth law (based on Ampere's circuital law)	$\oint H \cdot dl = \int_S (J + \frac{\partial D}{\partial t}) \bullet ds$	$\nabla \times \bar{B} = \mu_0 (j + \frac{\partial \bar{D}}{\partial t})$

Displacement current:

According to Maxwell Fourth law, steady current carrying conductor generates magnetic field around it.

$$\oint \bar{B}.\bar{dl} = \mu_0 i \qquad \rightarrow (1)$$

Hence

$$\oint B.dl = \mu_0 \int_S j \bullet ds \qquad \rightarrow (2)$$

Using Stoke's theorem $\oint B.dl = \int_S \nabla \times \bar{B} \bullet dS \quad \rightarrow (3)$

From (2) and (3) $\int_S \nabla \times \overline{B} \bullet dS = \mu_0 \int_S j \bullet ds$ →(4)

Or $\nabla \times \bar{B} = \mu_0 j$ → (5)

Taking divergence of this equation we get

$\nabla.(\nabla\times B)$ = div(curl B) = div$\mu_0 j$= μ_0j →(6)

However that divergence of curl of a vector is always zero and hence

div j = 0 →(7)

This shows that the total flux of current out of any closed surface is zero. However from equation of continuity

div j+$\frac{\partial \rho}{\partial t}$ =0 →(8)

The equation (8) contradicts equation(7)

From Maxwell I equation∇. D = ρ

Also div j+$\frac{\partial \rho}{\partial t}$ = div (j+$\frac{\partial D}{\partial t}$)

Thus ∇.j =0 for steady current

And ∇.(j+$\frac{\partial D}{\partial t}$) =0 every where

The term $\frac{\partial D}{\partial t}$ is called as displacement current density.

Thus a changing electric field is equivalent to a current which flows as long as the electric field is changing and produced the same magnetic effect as an ordinary conduction current. This is known as displacement current.

Electromagnetic wave equation:-

According to Maxwell's electromagnetic equations in a homogeneous medium

(i) It has infinite resistance to the current and hence its conductivity is zero i.e. **j** = 0

(ii) It has no volume distribution of charge, thus the charge density ρ =0

(iii) Also $\bar{D} = \epsilon \bar{E}$ and $\bar{B} = \mu \bar{H}$

Hence Maxwell equations

$\nabla \cdot \bar{E} = 0$ →(1)

$$\nabla \cdot \bar{B} = 0 \rightarrow (2)$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \rightarrow (3)$$

$$\nabla \times \bar{B} = \mu\varepsilon \frac{\partial \bar{E}}{\partial t} \rightarrow (4)$$

The wave equation of propagation of a wave can be obtained by taking curl of eq.(4) as

$$\nabla \times \nabla \times \bar{B} = \nabla \times \mu\varepsilon \frac{\partial \bar{E}}{\partial t} = \mu\varepsilon \left(\nabla \times \frac{\partial \bar{E}}{\partial t}\right)$$

$$= \mu\varepsilon \frac{\partial}{\partial t}(\nabla \times \bar{E}) \qquad (\because \text{ from eq.(3)})$$

$$= \mu\varepsilon \frac{\partial}{\partial t}\left(-\frac{\partial \bar{B}}{\partial t}\right)$$

$$= -\mu\varepsilon \frac{\partial^2 \bar{B}}{\partial t^2} \rightarrow (5)$$

Thus $\nabla \times \nabla \times \bar{B} = \nabla(\nabla . \bar{B}) - \nabla^2 \bar{B} = -\nabla^2 \bar{B} \rightarrow (6)$

From (5) and (6)

$$-\nabla^2 \bar{B} = -\mu\varepsilon \frac{\partial^2 \bar{B}}{\partial t^2}$$

Or $\nabla^2 \bar{B} = \mu\varepsilon \frac{\partial^2 \bar{B}}{\partial t^2} \rightarrow (7)$

Similarly, from eq.(3) we can show that

$$\nabla^2 \bar{E} = \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2} \rightarrow (8)$$

From eqs.(7) and (8) represents the relation between the space and tie variation of magnetic field $\bar{B}$ and electric field $\bar{E}$. Hence the general wave equation is represented by

$$\boxed{\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}} \rightarrow (9)$$

$$\frac{1}{v^2} = \mu\varepsilon \rightarrow (10)$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

Where μ and ε are permeability and permittivity of the medium

When the electromagnetic wave propagating in free space

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$\mu_0 = 4\pi\times10^{-7}$ and $\varepsilon_0 = 1/(4\pi\times9\times10^{9})$

$$v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{1}{4\pi \times 9 \times 10^{9}}}}$$

$$v = \sqrt{9 \times 10^{16}} = 3\times10^{8} \text{ m/s}$$

Thus the velocity of propagation of variation of **E** and **B** is the same as the velocity of light.

Equation (9) indicate the wave propagation in 3-D free space. These waves involve periodic variations of electric and magnetic field. So they are called electromagnetic waves.

Ultrasonic's: Ultrasonic sound waves have frequencies above the human ear's audible range that is greater than 20kHz and often into megahertz range.

<u>PRODUCTION OF ULTRASONIC WAVES:</u>

The ultrasonic waves cannot be produced by our usual method of loudspeaker fed with alternating current. This is due to the fact that at very high frequencies. There are two methods namely magnetostriction and piezoelectric are used to produce the ultrasonic's

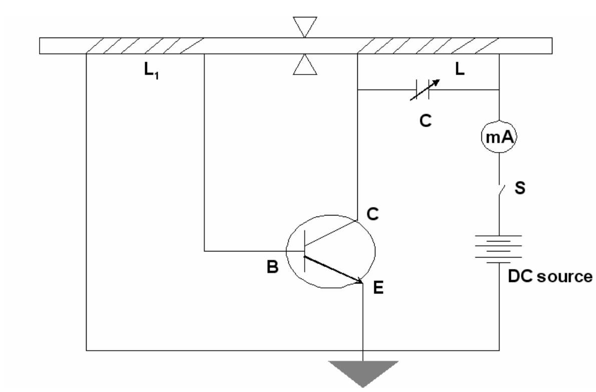
<u>Magnetostriction method</u>

Principle:

When an alternating magnetic field is applied parallel to the length of a ferromagnetic rod such as iron or nickel, a small elongation or contraction occurs in its length. This phenomenon is known as magnetostriction.

Construction:

An experimental arrangement due to production of ultrasonic waves is shown in fig. There is a short nickel rod which is clamped at the centre. A simple tuned oscillator constructed with a NPN transistor with L-C circuit is connected in the collector



Working:

When the supply is switched on, collector current starts rising and oscillations start in the L-C circuit, the changes of current in inductor L is feedback to the base emitter circuit through mutual inductance between L and L1. The transistor merely ensures that energy is feedback at the correct phase from the source. The frequency of oscillation of L-C circuit is given by $f = 1/2\pi\sqrt{(LC)}$. By varying C, the frequency can be adjusted to be in tune with the natural frequency of the rod. Under the resonance condition sustained oscillations and hence ultrasonic waves are produced by the rod.

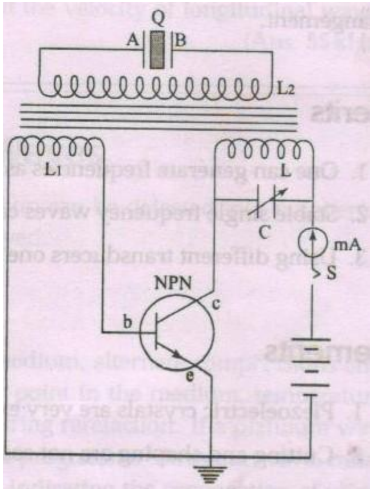
Piezoelectric method:

Principle:

In certain crystals such as quartz is subjected to pressure on one pair of opposite faces (mechanical faces) then in the other pair of opposite faces (electric faces), an opposite electric charges are developed. Similarly when the electric faces are subjected to the alternating voltage then the mechanical faces produces frequencies of order more than 100 kHz

Construction:

The circuit diagram used for generating ultrasonic waves using piezoelectric effect is shown in fig. Q is a thin slice of quartz crystal cut with its opposite faces perpendicular to optic axes. The crystal Q is placed between two metal plates A and B which act as electrodes. The plates are connected to the coil L_2 Coil L is connected to the collector circuit.



Working :

When the supply is switched on, collector current starts raising and oscillations start in the L-C circuit. The frequency of oscillation of the L-C circuit is given by the expression $f = 1/2\pi\sqrt{(LC)}$. By varying C the frequency of oscillation of the circuit can be adjusted such that electrodes A and B connected to the coil L_2 are induced with alternating e.m.f. Hence the crystal

Q placed between the electrodes A and B experience oscillating electric force and due to inverse piezoelectric effect, high frequency ultrasonic waves are produced.

APPLICATIONS:

1) Ultrasonic inspection is used for quality control and material applications
2) Used to measure thickness of metal sections
3) Thickness measurements are made on refinery and chemical procession equipments, submarine hulls, aircraft sections and pressure vessels.
4) Used to detect internal corrosion.